Flow2Vec: Value-Flow-Based Precise Code Embedding

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Contribution

A new code embedding approach which marries **precise static analysis** and recent advances in **high-order proximity embedding**, by preserving

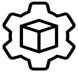
- **interprocedural alias-aware program dependence**, and
- **context-free language reachability**,

to better support subsequent code analysis tasks, such as code summarization and semantic labeling.

Code Embedding

Source Code | **Model** | **Code Property Prediction**

```
int* ____(int[] myArray, int size)
{
    for (int i = 0; i < size; i++)
        myArray[i] = i;
    return myArray;
}
```



Code Embedding

Source Code

Model

Semantic Label

```
int* ____(int[] myArray, int size)
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    for (int i = 0; i < size; i++)
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```
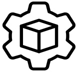
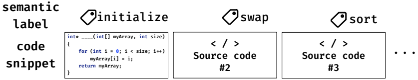



initialize

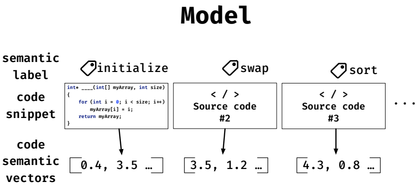
Code Embedding

Model



Code Embedding

Code semantic vector in geometric space



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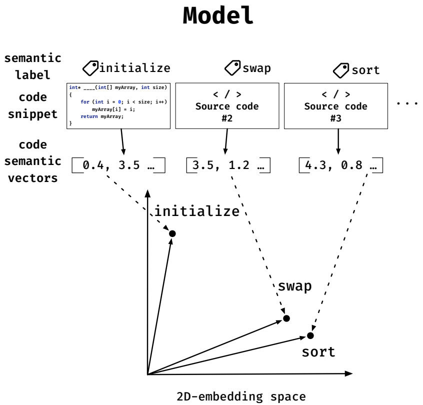

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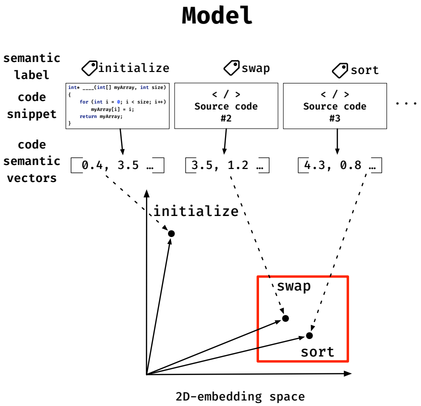

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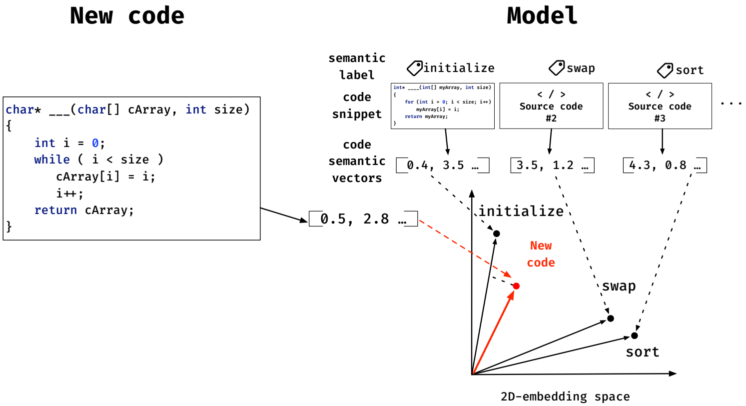

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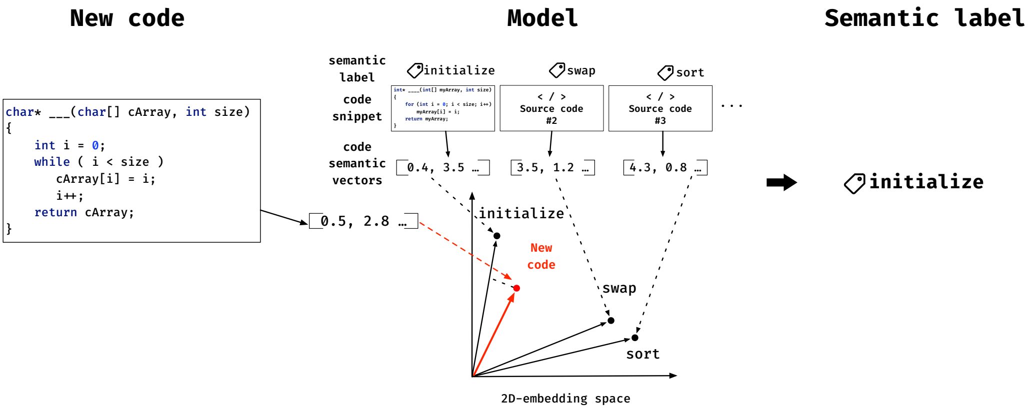

Existing Embedding Approaches

Structure-oblivious embedding

Source code ➡ **A bag of 'sentences'**[1,2]

```
int* ____(int[] myArray, int size)
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    for (int i = 0; i < size; i++)
        myArray[i] = i;
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```

[1] Distributed representations of words and phrases and their compositionality. In NeurIPS '13

[2] Distributed representations of sentences and documents. In ICML '14

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➡

```
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```

→

int	*	____	(	int	[]	myArray	,	int	size	)

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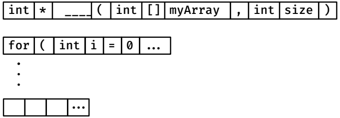

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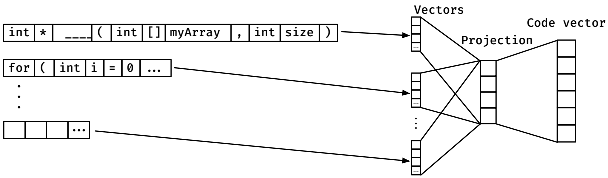

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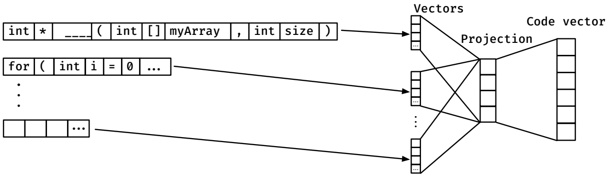
```
int* ____(int[] myArray, int size)
{
    for (int i = 0; i < size; i++)
        myArray[i] = i;
    return myArray;
}

char* ___(char[] cArray, int size)
{
    int i = 0;
    while ( i < size )
       cArray[i] = i;
       i++;
    return cArray;
}
```



Textually different but semantically equivalent

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Existing Embedding Approaches

Structure-preserving embedding

Source code

```
int* ____(int[] myArray, int size)
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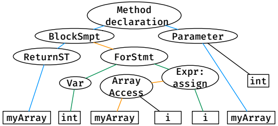
Abstract Syntax Tree [3]



[3] code2vec: Learning distributed representations of code. POPL .2019

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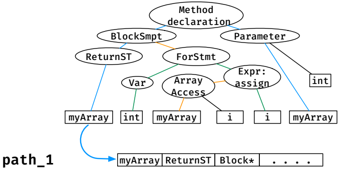
A bag of 'paths' on AST [3]



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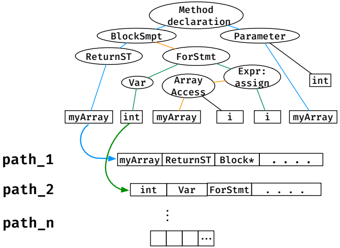

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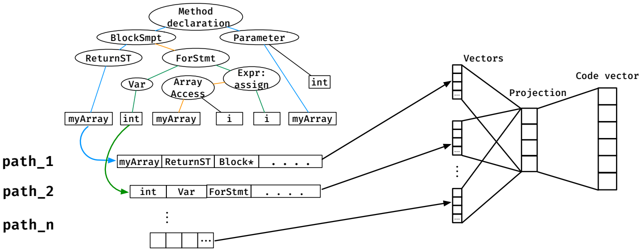

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Problems and Limitations

(a) Fail to capture asymmetric transitivity

(b) Alias-unaware

(c) Intraprocedural/context-insensitivity

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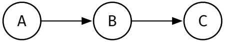

program dependence graph

(b) Alias-unaware

(c) Intraprocedural/context-insensitivity

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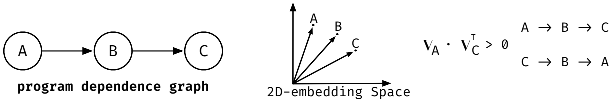

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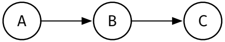

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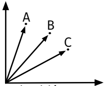

2D-embedding Space

$V_A \cdot V_C^T > 0$

A → B → C ✓ **Real reachability and correctly preserved**

C → B → A ✗ **Spurious reachability but imprecisely preserved**

(b) Alias-unaware

(c) Intraprocedural/context-insensitivity

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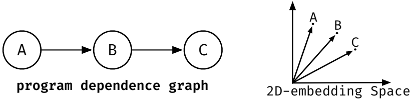

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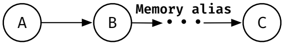

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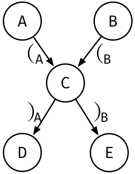

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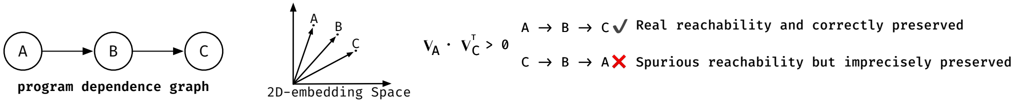

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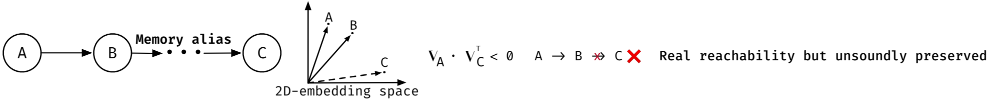

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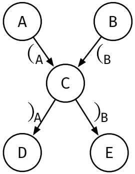

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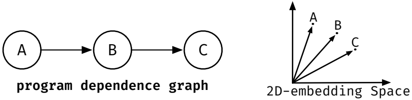

$V_A \cdot V_C^T > 0$

A → B → C ✔ Real reachability and correctly preserved

C → B → A ✘ Spurious reachability but imprecisely preserved

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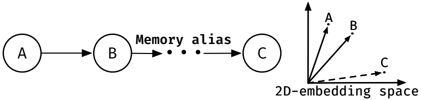

$V_A \cdot V_C^T < 0$ A → B ↛ C ✘ Real reachability but unsoundly preserved

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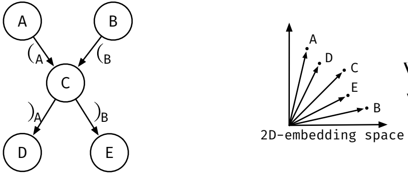
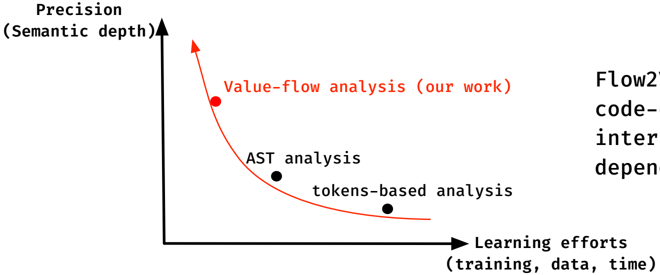


$V_A \cdot V_D^T > 0$ A → C → D ✔ Real reachability and correctly preserved

$V_A \cdot V_E^T > 0$ A → C → E ✘ Spurious reachability but imprecisely preserved

The Aim of This Work



Flow2Vec: a high-order proximity code-embedding approach by preserving interprocedural alias-aware program dependence

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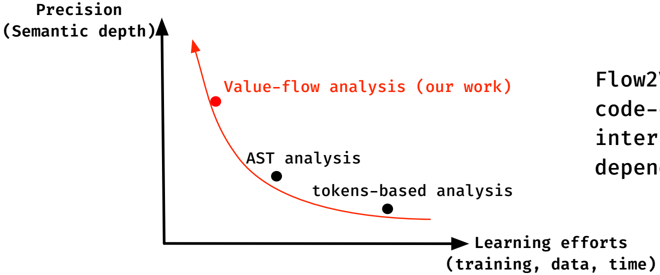

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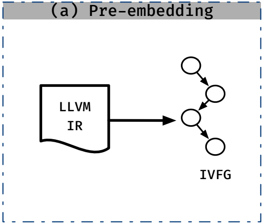

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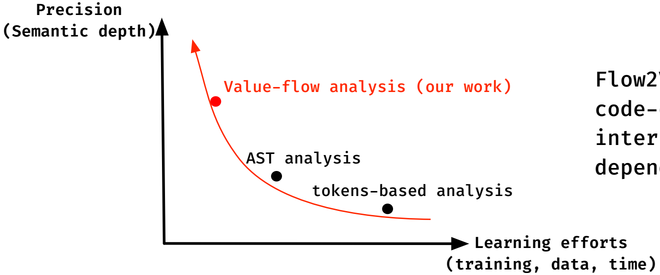

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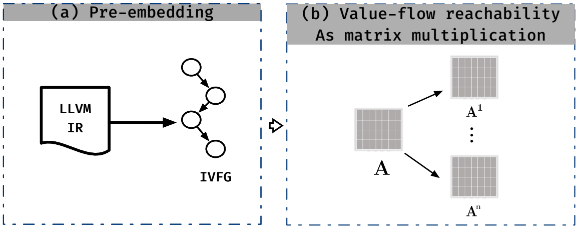

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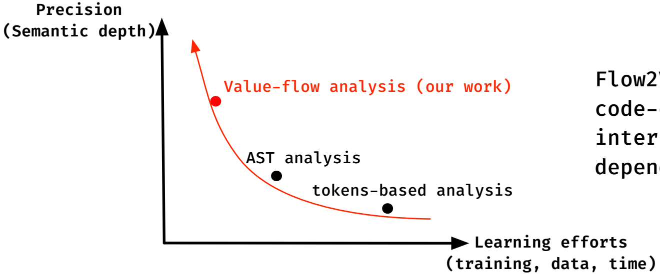

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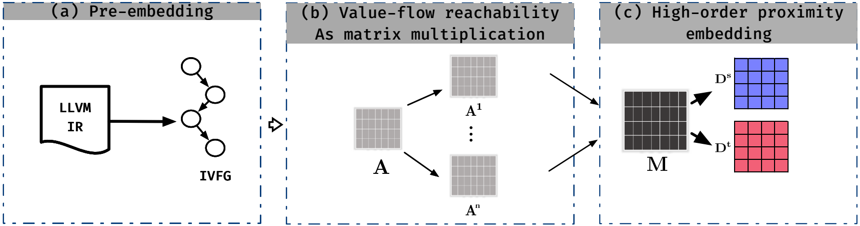

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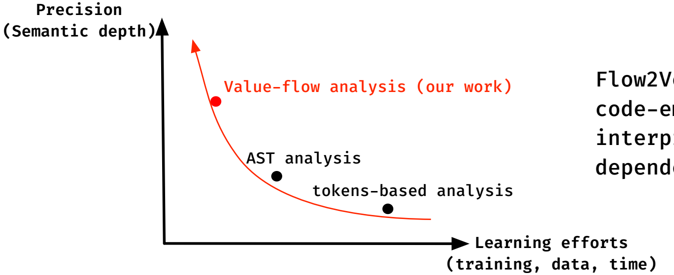

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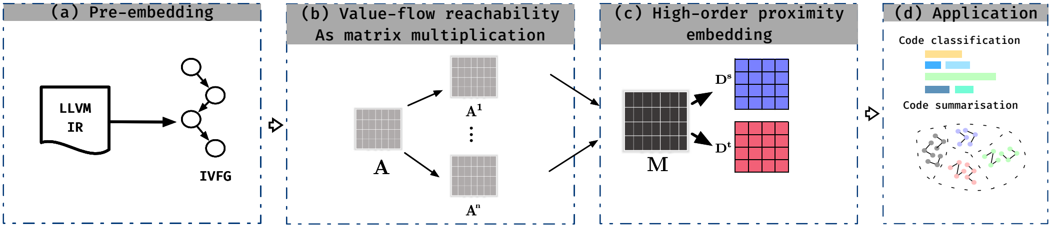

A Motivating Example

Phase (a) Pre-embedding

```
    foo(){
l1:   stack = malloc(...);
l2:   queue = malloc(...);
l3:   p = initialize(stack); // cs1
l4:   q = initialize(queue); // cs2


      ……
    }
l5: initialize(x){
      // initialization for
      // objects that 'x' points to
      ……
l6:   return x;
    }
```

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Phase (a) Pre-embedding

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    foo(){
l1:   stack = malloc(...);
l2:   queue = malloc(...);
l3:   p = initialize(stack); // cs1
l4:   q = initialize(queue); // cs2
      // operations on 'stack'
      // via pointer 'p'
      ......
    }
l5: initialize(x){
      // initialization for
      // objects that 'x' points to
      ......
l6:   return x;
    }
```
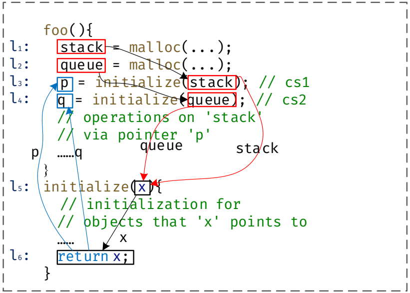

Call site 1	p refers to stack
Call site 2	q refers to queue

A Motivating Example

Phase (a) Pre-embedding



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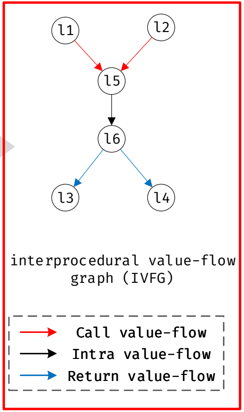
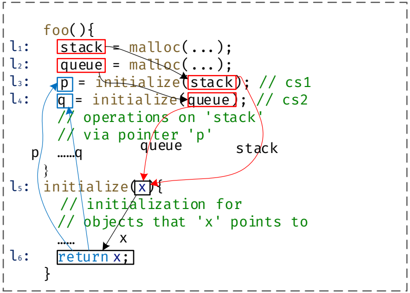
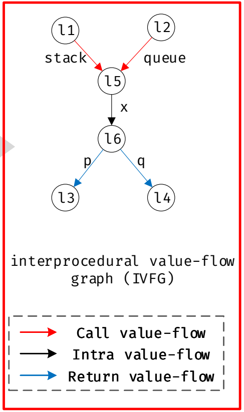


A Motivating Example

Phase (a) Pre-embedding





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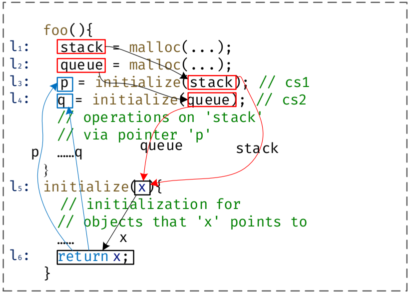

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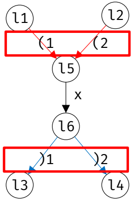

interprocedural value-flow graph (IVFG)

- Call value-flow
- Intra value-flow
- Return value-flow

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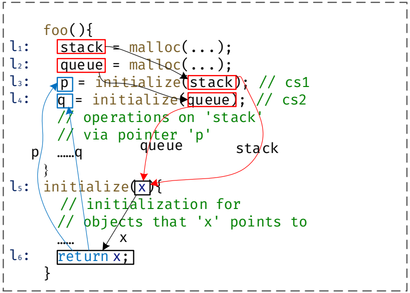

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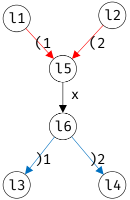


interprocedural value-flow graph (IVFG)

→	Call value-flow
→	Intra value-flow
→	Return value-flow

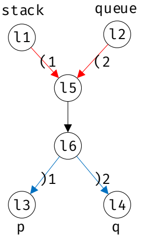
$$
\begin{array}{c} \\ l_1 \\ l_2 \\ l_3 \\ l_4 \\ l_5 \\ l_6 \end{array}
\begin{array}{c}
\begin{array}{cccccc} l_1 & l_2 & l_3 & l_4 & l_5 & l_6 \end{array} \\
\left[\begin{array}{cccccc}
0 & 0 & 0 & 0 & (_1 & 0 \\
0 & 0 & 0 & 0 & (_2 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 &)_1 &)_2 & 0 & 0
\end{array}\right]
\end{array}
$$

$\mathbf{A}$

Symbolic Adjacency matrix

A Motivating Example

Phase (b) Value-flow reachability as matrix multiplication



IVFG

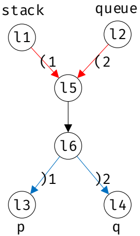
$$
\begin{array}{c}
 \\ l_1 \\ l_2 \\ l_3 \\ l_4 \\ l_5 \\ l_6
\end{array}
\begin{array}{c}
\begin{array}{cccccc} l_1 & l_2 & l_3 & l_4 & l_5 & l_6 \end{array} \\
\begin{bmatrix}
0 & 0 & 0 & 0 & (_1 & 0 \\
0 & 0 & 0 & 0 & (_2 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 &)_1 &)_2 & 0 & 0
\end{bmatrix}
\end{array}
$$

$\mathbf{A}$

Symbolic Adjacency matrix

A Motivating Example

Phase (b) Value-flow reachability as matrix multiplication



IVFG

$$\begin{array}{c|c} & \begin{matrix} l_1 & l_2 & l_3 & l_4 & l_5 & l_6 \end{matrix} \\ \begin{matrix} l_1 \\ l_2 \\ l_3 \\ l_4 \\ l_5 \\ l_6 \end{matrix} & \begin{bmatrix} 0 & 0 & 0 & 0 & (_1 & 0 \\ 0 & 0 & 0 & 0 & (_2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 &)_1 &)_2 & 0 & 0 \end{bmatrix} \end{array}$$

$\mathbf{A}$

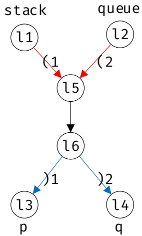
Symbolic Adjacency matrix

$$\mathbf{A}^h = \underbrace{\mathbf{A} \cdot \ldots \cdot \mathbf{A}}_{\times h}$$

The power of matrix
h-th order reachability

A Motivating Example

Phase (b) Value-flow reachability as matrix multiplication



IVFG

$$\begin{array}{c|c} & \begin{array}{cccccc} l_1 & l_2 & l_3 & l_4 & l_5 & l_6 \end{array} \\ \begin{array}{c} l_1 \\ l_2 \\ l_3 \\ l_4 \\ l_5 \\ l_6 \end{array} & \begin{bmatrix} 0 & 0 & 0 & 0 & (_1 & 0 \\ 0 & 0 & 0 & 0 & (_2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 &)_1 &)_2 & 0 & 0 \end{bmatrix} \end{array}$$

$\mathbf{A}$

Symbolic Adjacency matrix

$$\begin{array}{c|c} & \begin{array}{cccccc} l_1 & l_2 & l_3 & l_4 & l_5 & l_6 \end{array} \\ \begin{array}{c} l_1 \\ l_2 \\ l_3 \\ l_4 \\ l_5 \\ l_6 \end{array} & \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & (_1 * 1 \\ 0 & 0 & 0 & 0 & 0 & (_2 * 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 *)_1 & 1 *)_2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

$\mathbf{A}^2 = \mathbf{A} \cdot \mathbf{A}$

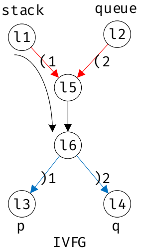
Matrix multiplication

$$\mathbf{A}^h = \underbrace{\mathbf{A} \cdot \ldots \cdot \mathbf{A}}_{\times h}$$

The power of matrix
h-th order reachability

A Motivating Example

Phase (b) Value-flow reachability as matrix multiplication



$$
\begin{array}{c}
 \\ l_1 \\ l_2 \\ l_3 \\ l_4 \\ l_5 \\ l_6
\end{array}
\begin{array}{c}
\begin{array}{cccccc} l_1 & l_2 & l_3 & l_4 & l_5 & l_6 \end{array} \\
\begin{bmatrix}
0 & 0 & 0 & 0 & (_1 & 0 \\
0 & 0 & 0 & 0 & (_2 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 &)_1 &)_2 & 0 & 0
\end{bmatrix}
\end{array}
$$

$\mathbf{A}$

Symbolic Adjacency matrix

$$
\begin{array}{c}
 \\ l_1 \\ l_2 \\ l_3 \\ l_4 \\ l_5 \\ l_6
\end{array}
\begin{array}{c}
\begin{array}{cccccc} l_1 & l_2 & l_3 & l_4 & l_5 & l_6 \end{array} \\
\begin{bmatrix}
0 & 0 & 0 & 0 & 0 & \boxed{(_1 * 1} \\
0 & 0 & 0 & 0 & 0 & (_2 * 1 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 *)_1 & 1 *)_2 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\end{array}
$$

$$\mathbf{A}^2 = \mathbf{A} \cdot \mathbf{A}$$

Matrix multiplication

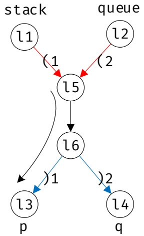
$$\boxed{\mathbf{A}^2_{1,6} = (_1 * 1 \quad l_1 \xrightarrow{(_1} l_5 \rightarrow l_6}$$

$$\mathbf{A}^h = \underbrace{\mathbf{A} \cdot \ldots \cdot \mathbf{A}}_{\times h}$$

The power of matrix
h-th order reachability

A Motivating Example

Phase (b) Value-flow reachability as matrix multiplication



IVFG

$$
\begin{array}{c|c}
 & \begin{matrix} l_1 & l_2 & l_3 & l_4 & l_5 & l_6 \end{matrix} \\
\begin{matrix} l_1 \\ l_2 \\ l_3 \\ l_4 \\ l_5 \\ l_6 \end{matrix} &
\begin{bmatrix}
0 & 0 & 0 & 0 & (_1 & 0 \\
0 & 0 & 0 & 0 & (_2 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 &)_1 &)_2 & 0 & 0
\end{bmatrix}
\end{array}
$$

$\mathbf{A}$

Symbolic Adjacency matrix

$$
\begin{array}{c|c}
 & \begin{matrix} l_1 & l_2 & l_3 & l_4 & l_5 & l_6 \end{matrix} \\
\begin{matrix} l_1 \\ l_2 \\ l_3 \\ l_4 \\ l_5 \\ l_6 \end{matrix} &
\begin{bmatrix}
0 & 0 & 0 & 0 & 0 & (_1{*}1 \\
0 & 0 & 0 & 0 & 0 & (_2{*}1 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \boxed{1{*})_1} & 1{*})_2 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\end{array}
$$

$\mathbf{A}^2 = \mathbf{A} \cdot \mathbf{A}$

Matrix multiplication

$\mathbf{A}^2_{1,6} = (_1{*}1 \quad l_1 \xrightarrow{(_1} l_5 \rightarrow l_6$

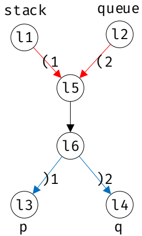
$\boxed{\mathbf{A}^2_{5,3} = 1{*})_1 \quad l_5 \rightarrow l_6 \xrightarrow{)_1} l_3}$

$$\mathbf{A}^h = \underbrace{\mathbf{A} \cdot \ldots \cdot \mathbf{A}}_{\times h}$$

The power of matrix
h-th order reachability

A Motivating Example

Phase (b) Value-flow reachability as matrix multiplication



IVFG

$$
\begin{array}{c|cccccc}
 & l_1 & l_2 & l_3 & l_4 & l_5 & l_6 \\
\hline
l_1 & 0 & 0 & 0 & 0 & (_1 & 0 \\
l_2 & 0 & 0 & 0 & 0 & (_2 & 0 \\
l_3 & 0 & 0 & 0 & 0 & 0 & 0 \\
l_4 & 0 & 0 & 0 & 0 & 0 & 0 \\
l_5 & 0 & 0 & 0 & 0 & 0 & 1 \\
l_6 & 0 & 0 &)_1 &)_2 & 0 & 0
\end{array}
$$

$\mathbf{A}$

Symbolic Adjacency matrix

$$
\begin{array}{c|cccccc}
 & l_1 & l_2 & l_3 & l_4 & l_5 & l_6 \\
\hline
l_1 & 0 & 0 & (_1{*}1{*})_1 & (_1{*}1{*})_2 & 0 & 0 \\
l_2 & 0 & 0 & (_2{*}1{*})_1 & (_2{*}1{*})_2 & 0 & 0 \\
l_3 & 0 & 0 & 0 & 0 & 0 & 0 \\
l_4 & 0 & 0 & 0 & 0 & 0 & 0 \\
l_5 & 0 & 0 & 0 & 0 & 0 & 0 \\
l_6 & 0 & 0 & 0 & 0 & 0 & 0
\end{array}
$$

$\mathbf{A}^3 = \mathbf{A} \cdot \mathbf{A} \cdot \mathbf{A}$

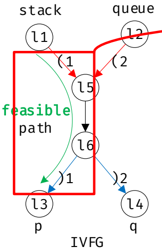
Matrix multiplication

$$\mathbf{A}^h = \underbrace{\mathbf{A} \cdot \ldots \cdot \mathbf{A}}_{\times h}$$

The power of matrix
h-th order reachability

A Motivating Example

Phase (b) Value-flow reachability as matrix multiplication



$$
\begin{array}{c}
\begin{array}{cccccc} l_1 & l_2 & l_3 & l_4 & l_5 & l_6 \end{array} \\
\begin{array}{c} l_1 \\ l_2 \\ l_3 \\ l_4 \\ l_5 \\ l_6 \end{array}
\begin{bmatrix}
0 & 0 & 0 & 0 & (_1 & 0 \\
0 & 0 & 0 & 0 & (_2 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 &)_1 &)_2 & 0 & 0
\end{bmatrix}
\end{array}
$$

$\mathbf{A}$

Symbolic Adjacency matrix

$$
\begin{array}{c}
\begin{array}{cccccc} l_1 & l_2 & l_3 & l_4 & l_5 & l_6 \end{array} \\
\begin{array}{c} l_1 \\ l_2 \\ l_3 \\ l_4 \\ l_5 \\ l_6 \end{array}
\begin{bmatrix}
0 & 0 & \boxed{(_1 {*} 1 {*})_1} & (_1 {*} 1 {*})_2 & 0 & 0 \\
0 & 0 & (_2 {*} 1 {*})_1 & (_2 {*} 1 {*})_2 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\end{array}
$$

$\mathbf{A}^3 = \mathbf{A} \cdot \mathbf{A} \cdot \mathbf{A}$ Matrix multiplication

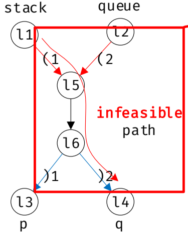
$\mathbf{A}^3_{1,3} = (_1 {*} 1 {*})_1$ $\quad l_1 \xrightarrow{(_1} l_5 \rightarrow l_6 \xrightarrow{)_1} l_3$ feasible path of length 3

$$\mathbf{A}^h = \underbrace{\mathbf{A} \cdot \ldots \cdot \mathbf{A}}_{\times h}$$

The power of matrix
h-th order reachability

A Motivating Example

Phase (b) Value-flow reachability as matrix multiplication



$$
\begin{array}{c|cccccc}
 & l_1 & l_2 & l_3 & l_4 & l_5 & l_6 \\
\hline
l_1 & 0 & 0 & 0 & 0 & (_1 & 0 \\
l_2 & 0 & 0 & 0 & 0 & (_2 & 0 \\
l_3 & 0 & 0 & 0 & 0 & 0 & 0 \\
l_4 & 0 & 0 & 0 & 0 & 0 & 0 \\
l_5 & 0 & 0 & 0 & 0 & 0 & 1 \\
l_6 & 0 & 0 &)_1 &)_2 & 0 & 0
\end{array}
$$

$\mathbf{A}$

Symbolic Adjacency matrix

$$
\begin{array}{c|cccccc}
 & l_1 & l_2 & l_3 & l_4 & l_5 & l_6 \\
\hline
l_1 & 0 & 0 & (_1{*}1{*})_1 & \boxed{(_1{*}1{*})_2} & 0 & 0 \\
l_2 & 0 & 0 & (_2{*}1{*})_1 & (_2{*}1{*})_2 & 0 & 0 \\
l_3 & 0 & 0 & 0 & 0 & 0 & 0 \\
l_4 & 0 & 0 & 0 & 0 & 0 & 0 \\
l_5 & 0 & 0 & 0 & 0 & 0 & 0 \\
l_6 & 0 & 0 & 0 & 0 & 0 & 0
\end{array}
$$

$\mathbf{A}^3 = \mathbf{A} \cdot \mathbf{A} \cdot \mathbf{A}$ Matrix multiplication

$\mathbf{A}^3_{1,3} = (_1{*}1{*})_1 \quad l1 \xrightarrow{(1} l5 \rightarrow l6 \xrightarrow{)1} l3$

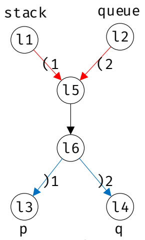
$\mathbf{A}^3_{1,4} = (_1{*}1{*})_2 \quad l1 \xrightarrow{(1} l5 \rightarrow l6 \xrightarrow{)2} l4$ infeasible path of length 3

$$\mathbf{A}^h = \underbrace{\mathbf{A} \cdot \ldots \cdot \mathbf{A}}_{\times h}$$

The power of matrix
h-th order reachability

A Motivating Example

Phase (b) Value-flow reachability as matrix multiplication



IVFG

$$
\begin{array}{c}
 \\ l_1 \\ l_2 \\ l_3 \\ l_4 \\ l_5 \\ l_6
\end{array}
\begin{array}{c}
\begin{array}{cccccc} l_1 & l_2 & l_3 & l_4 & l_5 & l_6 \end{array} \\
\begin{bmatrix}
0 & 0 & 0 & 0 & (_1 & 0 \\
0 & 0 & 0 & 0 & (_2 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 &)_1 &)_2 & 0 & 0
\end{bmatrix}
\end{array}
$$

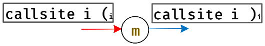
$\mathbf{A}$

Symbolic Adjacency matrix

$$
\begin{array}{c|cccccc}
 & l_1 & l_2 & l_3 & l_4 & l_5 & l_6 \\
\hline
l_1 & 0 & 0 & \boxed{(_1 1^* 1^*)_1} & \cancel{(_1 1^* 1^*)_2} & 0 & 0 \\
l_2 & 0 & 0 & \cancel{(_2 1^* 1^*)_1} & \boxed{(_2 1^* 1^*)_2} & 0 & 0 \\
l_3 & 0 & 0 & 0 & 0 & 0 & 0 \\
l_4 & 0 & 0 & 0 & 0 & 0 & 0 \\
l_5 & 0 & 0 & 0 & 0 & 0 & 0 \\
l_6 & 0 & 0 & 0 & 0 & 0 & 0
\end{array}
$$

$\mathbf{A}^3 = \mathbf{A} \cdot \mathbf{A} \cdot \mathbf{A}$ Matrix multiplication

$L : C = CC \mid (_1\ C\ )_1 \mid \ldots \mid (_n\ C\ )_n \mid \epsilon$ context-free language

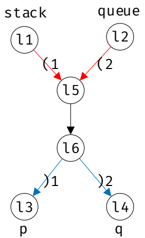


entering a method m from $\boxed{\text{callsite i } (_i}$

and exits from m back to $\boxed{\text{callsite i })_i}$

A Motivating Example

Phase (b) Value-flow reachability as matrix multiplication



IVFG

$$
\begin{array}{c|cccccc}
 & l_1 & l_2 & l_3 & l_4 & l_5 & l_6 \\
\hline
l_1 & 0 & 0 & 0 & 0 & (_1 & 0 \\
l_2 & 0 & 0 & 0 & 0 & (_2 & 0 \\
l_3 & 0 & 0 & 0 & 0 & 0 & 0 \\
l_4 & 0 & 0 & 0 & 0 & 0 & 0 \\
l_5 & 0 & 0 & 0 & 0 & 0 & 1 \\
l_6 & 0 & 0 &)_1 &)_2 & 0 & 0
\end{array}
$$

$\mathbf{A}$

Symbolic Adjacency matrix

$$
\begin{array}{c|cccccc}
 & l_1 & l_2 & l_3 & l_4 & l_5 & l_6 \\
\hline
l_1 & 0 & 0 & \boxed{1} & \boxed{0} & 0 & 0 \\
l_2 & 0 & 0 & \boxed{0} & \boxed{1} & 0 & 0 \\
l_3 & 0 & 0 & 0 & 0 & 0 & 0 \\
l_4 & 0 & 0 & 0 & 0 & 0 & 0 \\
l_5 & 0 & 0 & 0 & 0 & 0 & 0 \\
l_6 & 0 & 0 & 0 & 0 & 0 & 0
\end{array}
$$

$$\mathbf{A}^3 = \mathbf{A} \cdot \mathbf{A} \cdot \mathbf{A}$$

Matrix multiplication

A Motivating Example

Phase (c) High-order proximity embedding

$$
\begin{array}{c|cccccc}
 & l_1 & l_2 & l_3 & l_4 & l_5 & l_6 \\
\hline
l_1 & 0 & 0 & 0 & 0 & (_1 & 0 \\
l_2 & 0 & 0 & 0 & 0 & (_2 & 0 \\
l_3 & 0 & 0 & 0 & 0 & 0 & 0 \\
l_4 & 0 & 0 & 0 & 0 & 0 & 0 \\
l_5 & 0 & 0 & 0 & 0 & 0 & 1 \\
l_6 & 0 & 0 &)_1 &)_2 & 0 & 0
\end{array}
$$

$\mathbf{A}$

$$
\begin{array}{c|cccccc}
 & l_1 & l_2 & l_3 & l_4 & l_5 & l_6 \\
\hline
l_1 & 0 & 0 & 0 & 0 & 0 & (_1{*}1 \\
l_2 & 0 & 0 & 0 & 0 & 0 & (_2{*}1 \\
l_3 & 0 & 0 & 0 & 0 & 0 & 0 \\
l_4 & 0 & 0 & 0 & 0 & 0 & 0 \\
l_5 & 0 & 0 & 1{*})_1 & 1{*})_2 & 0 & 0 \\
l_6 & 0 & 0 & 0 & 0 & 0 & 0
\end{array}
$$

$\mathbf{A}^2 = \mathbf{A} \cdot \mathbf{A}$

$$
\begin{array}{c|cccccc}
 & l_1 & l_2 & l_3 & l_4 & l_5 & l_6 \\
\hline
l_1 & 0 & 0 & (_1{*}1{*})_1 & (_1{*}1{*})_2 & 0 & 0 \\
l_2 & 0 & 0 & (_2{*}1{*})_1 & (_2{*}1{*})_2 & 0 & 0 \\
l_3 & 0 & 0 & 0 & 0 & 0 & 0 \\
l_4 & 0 & 0 & 0 & 0 & 0 & 0 \\
l_5 & 0 & 0 & 0 & 0 & 0 & 0 \\
l_6 & 0 & 0 & 0 & 0 & 0 & 0
\end{array}
$$

$\mathbf{A}^3 = \mathbf{A} \cdot \mathbf{A} \cdot \mathbf{A}$

Katz Index $\mathbf{M} = \sum_{h=1}^{H} (\beta \cdot \mathbf{A})^h = 0.8 \cdot \mathbf{A} + 0.8^2 \cdot \mathbf{A}^2 + 0.8^3 \cdot \mathbf{A}^3$

A Motivating Example

Phase (c) High-order proximity embedding

$$
\begin{array}{c|cccccc}
 & l_1 & l_2 & l_3 & l_4 & l_5 & l_6 \\
\hline
l_1 & 0 & 0 & 0 & 0 & (_1 & 0 \\
l_2 & 0 & 0 & 0 & 0 & (_2 & 0 \\
l_3 & 0 & 0 & 0 & 0 & 0 & 0 \\
l_4 & 0 & 0 & 0 & 0 & 0 & 0 \\
l_5 & 0 & 0 & 0 & 0 & 0 & 1 \\
l_6 & 0 & 0 &)_1 &)_2 & 0 & 0
\end{array}
$$

$\mathbf{A}$

$$
\begin{array}{c|cccccc}
 & l_1 & l_2 & l_3 & l_4 & l_5 & l_6 \\
\hline
l_1 & 0 & 0 & 0 & 0 & 0 & (_1*1 \\
l_2 & 0 & 0 & 0 & 0 & 0 & (_2*1 \\
l_3 & 0 & 0 & 0 & 0 & 0 & 0 \\
l_4 & 0 & 0 & 0 & 0 & 0 & 0 \\
l_5 & 0 & 0 & 1*)_1 & 1*)_2 & 0 & 0 \\
l_6 & 0 & 0 & 0 & 0 & 0 & 0
\end{array}
$$

$\mathbf{A}^2 = \mathbf{A} \cdot \mathbf{A}$

$$
\begin{array}{c|cccccc}
 & l_1 & l_2 & l_3 & l_4 & l_5 & l_6 \\
\hline
l_1 & 0 & 0 & (_1*1*)_1 & (_1*1*)_2 & 0 & 0 \\
l_2 & 0 & 0 & (_2*1*)_1 & (_2*1*)_2 & 0 & 0 \\
l_3 & 0 & 0 & 0 & 0 & 0 & 0 \\
l_4 & 0 & 0 & 0 & 0 & 0 & 0 \\
l_5 & 0 & 0 & 0 & 0 & 0 & 0 \\
l_6 & 0 & 0 & 0 & 0 & 0 & 0
\end{array}
$$

$\mathbf{A}^3 = \mathbf{A} \cdot \mathbf{A} \cdot \mathbf{A}$

Katz Index

$$\mathbf{M} = \sum_{h=1}^{H} (\beta \cdot \mathbf{A})^h = \boxed{0.8} \cdot \mathbf{A} + \boxed{0.8^2} \cdot \mathbf{A}^2 + \boxed{0.8^3} \cdot \mathbf{A}^3$$

Context-sensitive value-flow reachability matrix

$$
\begin{array}{c|cccccc}
 & l_1 & l_2 & l_3 & l_4 & l_5 & l_6 \\
\hline
l_1 & 0 & 0 & 0.512*(_1*1*)_1 & \cancel{0.512*(_1*1*)_2} & 0.8*(_1 & 0.64*(_1*1 \\
l_2 & 0 & 0 & \cancel{0.512*(_2*1*)_1} & 0.512*(_2*1*)_2 & 0.8*(_2 & 0.64*(_2*1 \\
l_3 & 0 & 0 & 0 & 0 & 0 & 0 \\
l_4 & 0 & 0 & 0 & 0 & 0 & 0 \\
l_5 & 0 & 0 & 0.64*1*)_1 & 0.64*1*)_2 & 0 & 0.8 \\
l_6 & 0 & 0 & 0.8*)_1 & 0.8*)_2 & 0 & 0
\end{array}
$$

A Motivating Example

Phase (c) High-order proximity embedding

$$
\begin{array}{c|cccccc}
 & l_1 & l_2 & l_3 & l_4 & l_5 & l_6 \\
\hline
l_1 & 0 & 0 & 0 & 0 & (_1 & 0 \\
l_2 & 0 & 0 & 0 & 0 & (_2 & 0 \\
l_3 & 0 & 0 & 0 & 0 & 0 & 0 \\
l_4 & 0 & 0 & 0 & 0 & 0 & 0 \\
l_5 & 0 & 0 & 0 & 0 & 0 & 1 \\
l_6 & 0 & 0 &)_1 &)_2 & 0 & 0
\end{array}
$$

$\mathbf{A}$

$$
\begin{array}{c|cccccc}
 & l_1 & l_2 & l_3 & l_4 & l_5 & l_6 \\
\hline
l_1 & 0 & 0 & 0 & 0 & 0 & (_1*1 \\
l_2 & 0 & 0 & 0 & 0 & 0 & (_2*1 \\
l_3 & 0 & 0 & 0 & 0 & 0 & 0 \\
l_4 & 0 & 0 & 0 & 0 & 0 & 0 \\
l_5 & 0 & 0 & 1*)_1 & 1*)_2 & 0 & 0 \\
l_6 & 0 & 0 & 0 & 0 & 0 & 0
\end{array}
$$

$\mathbf{A}^2 = \mathbf{A} \cdot \mathbf{A}$

$$
\begin{array}{c|cccccc}
 & l_1 & l_2 & l_3 & l_4 & l_5 & l_6 \\
\hline
l_1 & 0 & 0 & (_1*1*)_1 & (_1*1*)_2 & 0 & 0 \\
l_2 & 0 & 0 & (_2*1*)_1 & (_2*1*)_2 & 0 & 0 \\
l_3 & 0 & 0 & 0 & 0 & 0 & 0 \\
l_4 & 0 & 0 & 0 & 0 & 0 & 0 \\
l_5 & 0 & 0 & 0 & 0 & 0 & 0 \\
l_6 & 0 & 0 & 0 & 0 & 0 & 0
\end{array}
$$

$\mathbf{A}^3 = \mathbf{A} \cdot \mathbf{A} \cdot \mathbf{A}$

Katz Index

$$\mathbf{M} = \sum_{h=1}^{H} (\beta \cdot \mathbf{A})^h = 0.8 \cdot \mathbf{A} + 0.8^2 \cdot \mathbf{A}^2 + 0.8^3 \cdot \mathbf{A}^3$$

Context-sensitive value-flow reachability matrix

$$
\begin{array}{c|cccccc}
 & l_1 & l_2 & l_3 & l_4 & l_5 & l_6 \\
\hline
l_1 & 0 & 0 & 0.512 & 0 & 0.8 & 0.64 \\
l_2 & 0 & 0 & 0 & 0.512 & 0.8 & 0.64 \\
l_3 & 0 & 0 & 0 & 0 & 0 & 0 \\
l_4 & 0 & 0 & 0 & 0 & 0 & 0 \\
l_5 & 0 & 0 & 0.64 & 0.64 & 0 & 0.8 \\
l_6 & 0 & 0 & 0.8 & 0.8 & 0 & 0
\end{array}
$$

A Motivating Example

Phase (c) High-order proximity embedding

$$\begin{array}{c|cccccc} & l_1 & l_2 & l_3 & l_4 & l_5 & l_6 \\ \hline l_1 & 0 & 0 & 0 & 0 & (_1 & 0 \\ l_2 & 0 & 0 & 0 & 0 & (_2 & 0 \\ l_3 & 0 & 0 & 0 & 0 & 0 & 0 \\ l_4 & 0 & 0 & 0 & 0 & 0 & 0 \\ l_5 & 0 & 0 & 0 & 0 & 0 & 1 \\ l_6 & 0 & 0 &)_1 &)_2 & 0 & 0 \end{array}$$

$$\mathbf{A}$$

$$\begin{array}{c|cccccc} & l_1 & l_2 & l_3 & l_4 & l_5 & l_6 \\ \hline l_1 & 0 & 0 & 0 & 0 & 0 & (_1 * 1 \\ l_2 & 0 & 0 & 0 & 0 & 0 & (_2 * 1 \\ l_3 & 0 & 0 & 0 & 0 & 0 & 0 \\ l_4 & 0 & 0 & 0 & 0 & 0 & 0 \\ l_5 & 0 & 0 & 1 *)_1 & 1 *)_2 & 0 & 0 \\ l_6 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}$$

$$\mathbf{A}^2 = \mathbf{A} \cdot \mathbf{A}$$

$$\begin{array}{c|cccccc} & l_1 & l_2 & l_3 & l_4 & l_5 & l_6 \\ \hline l_1 & 0 & 0 & (_1 * 1 *)_1 & (_1 * 1 *)_2 & 0 & 0 \\ l_2 & 0 & 0 & (_2 * 1 *)_1 & (_2 * 1 *)_2 & 0 & 0 \\ l_3 & 0 & 0 & 0 & 0 & 0 & 0 \\ l_4 & 0 & 0 & 0 & 0 & 0 & 0 \\ l_5 & 0 & 0 & 0 & 0 & 0 & 0 \\ l_6 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}$$

$$\mathbf{A}^3 = \mathbf{A} \cdot \mathbf{A} \cdot \mathbf{A}$$

Katz Index

$$\mathbf{M} = \sum_{h=1}^{H} (\beta \cdot \mathbf{A})^h = 0.8 \cdot \mathbf{A} + 0.8^2 \cdot \mathbf{A}^2 + 0.8^3 \cdot \mathbf{A}^3$$

Context-sensitive value-flow reachability matrix

$$\begin{array}{c|cccccc} & l_1 & l_2 & l_3 & l_4 & l_5 & l_6 \\ \hline l_1 & 0 & 0 & \boxed{0.512} & 0 & 0.8 & 0.64 \\ l_2 & 0 & 0 & 0 & 0.512 & 0.8 & 0.64 \\ l_3 & 0 & 0 & 0 & 0 & 0 & 0 \\ l_4 & 0 & 0 & 0 & 0 & 0 & 0 \\ l_5 & 0 & 0 & 0.64 & 0.64 & 0 & 0.8 \\ l_6 & 0 & 0 & 0.8 & 0.8 & 0 & 0 \end{array}$$

Example: l_1 to l_3

$$l1 \xrightarrow{(_1} l5 \rightarrow l6 \xrightarrow{)_1} l3$$

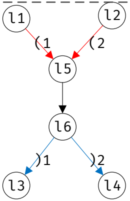
Reachability from l1 to l3:

$$\mathbf{M}_{1,3} = 0.8 \cdot \mathbf{A}_{1,3} + 0.8^2 \cdot \mathbf{A}^2_{1,3} + 0.8^3 \cdot \boxed{\mathbf{A}^3_{1,3}}$$

$$= 0.8 * 0 + 0.8^2 * 0 + 0.8^3 * \boxed{1} = 0.512$$

A Motivating Example

Phase (c) High-order proximity embedding



IVFG

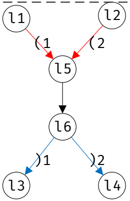
$$
\begin{array}{c|c}
 & \begin{matrix} l_1 & l_2 & l_3 & l_4 & l_5 & l_6 \end{matrix} \\
\begin{matrix} l_1 \\ l_2 \\ l_3 \\ l_4 \\ l_5 \\ l_6 \end{matrix} &
\begin{bmatrix}
0 & 0 & 0.512 & 0 & 0.8 & 0.64 \\
0 & 0 & 0 & 0.512 & 0.8 & 0.64 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0.64 & 0.64 & 0 & 0.8 \\
0 & 0 & 0.8 & 0.8 & 0 & 0
\end{bmatrix}
\end{array}
$$

$\mathbf{M}$

High-order proximity Matrix

A Motivating Example

Phase (c) High-order proximity embedding



IVFG

$$
\begin{array}{c}
 \\ l_1 \\ l_2 \\ l_3 \\ l_4 \\ l_5 \\ l_6
\end{array}
\begin{array}{c}
\begin{array}{cccccc} l_1 & l_2 & l_3 & l_4 & l_5 & l_6 \end{array} \\
\begin{bmatrix}
0 & 0 & 0.512 & 0 & 0.8 & 0.64 \\
0 & 0 & 0 & 0.512 & 0.8 & 0.64 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0.64 & 0.64 & 0 & 0.8 \\
0 & 0 & 0.8 & 0.8 & 0 & 0
\end{bmatrix}
\end{array}
$$

$\mathbf{M}$

High-order proximity Matrix

Singular Value Decompose

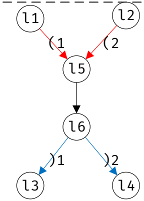
$$\mathbf{M} \approx \mathbf{U\Sigma V}^\top = \sum_{i=1}^{N} \sigma_i \mathbf{u}_i \mathbf{v}_i^\top = \mathbf{D}^s \cdot \mathbf{D}^{t\top}$$

$$\mathbf{D}^s = \left[\sqrt{\sigma_1} \cdot \mathbf{u}_1, \cdots, \sqrt{\sigma_K} \cdot \mathbf{u}_K\right] = \left[\mathbf{d}_1^s, \cdots, \mathbf{d}_N^s\right]$$

$$\mathbf{D}^t = \left[\sqrt{\sigma_1} \cdot \mathbf{v}_1, \cdots, \sqrt{\sigma_K} \cdot \mathbf{v}_K\right] = \left[\mathbf{d}_1^t, \cdots, \mathbf{d}_N^t\right]$$

A Motivating Example

Phase (c) High-order proximity embedding



IVFG

$$
\begin{array}{c|cccccc}
 & l_1 & l_2 & l_3 & l_4 & l_5 & l_6 \\
\hline
l_1 & 0 & 0 & 0.512 & 0 & 0.8 & 0.64 \\
l_2 & 0 & 0 & 0 & 0.512 & 0.8 & 0.64 \\
l_3 & 0 & 0 & 0 & 0 & 0 & 0 \\
l_4 & 0 & 0 & 0 & 0 & 0 & 0 \\
l_5 & 0 & 0 & 0.64 & 0.64 & 0 & 0.8 \\
l_6 & 0 & 0 & 0.8 & 0.8 & 0 & 0
\end{array}
$$

$$\mathbf{M}$$

High-order proximity Matrix

Singular Value Decompose

$$\mathbf{M} \approx \mathbf{U\Sigma V}^\top = \sum_{i=1}^{N} \sigma_i \mathbf{u}_i \mathbf{v}_i^\top = \mathbf{D}^s \cdot \mathbf{D}^{t\top}$$

$$\mathbf{D}^s = \left[\sqrt{\sigma_1}\cdot\mathbf{u}_1, \cdots, \sqrt{\sigma_K}\cdot\mathbf{u}_K\right] = \left[\mathbf{d}_1^s, \cdots, \mathbf{d}_N^s\right]$$

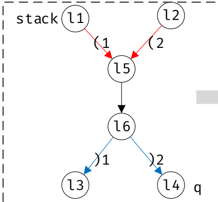
$$\mathbf{D}^t = \left[\sqrt{\sigma_1}\cdot\mathbf{v}_1, \cdots, \sqrt{\sigma_K}\cdot\mathbf{v}_K\right] = \left[\mathbf{d}_1^t, \cdots, \mathbf{d}_N^t\right]$$

$$
\left[\begin{array}{ccc|ccc}
-0.51 & -0.49 & -0.69 & 0 & 0 & 0 \\
0.51 & -0.49 & -0.69 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0.33 & -0.79 & 0 & 0 & 0 \\
0 & 0.71 & -0.58 & 0 & 0 & 0
\end{array}\right]
\qquad
\left[\begin{array}{ccc|ccc}
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
-0.51 & 0.49 & -0.69 & 0 & 0 & 0 \\
0.51 & 0.49 & -0.69 & 0 & 0 & 0 \\
0 & -0.71 & -0.58 & 0 & 0 & 0 \\
0 & -0.33 & -0.79 & 0 & 0 & 0
\end{array}\right]
$$

$$\mathbf{D}^s \qquad\qquad \mathbf{D}^t$$

A Motivating Example

Phase (c) High-order proximity embedding



IVFG

$$
\begin{array}{c}
 \\ l_1 \\ l_2 \\ l_3 \\ l_4 \\ l_5 \\ l_6
\end{array}
\begin{array}{c}
\begin{array}{cccccc} l_1 & l_2 & l_3 & l_4 & l_5 & l_6 \end{array} \\
\begin{bmatrix}
0 & 0 & 0.512 & 0 & 0.8 & 0.64 \\
0 & 0 & 0 & 0.512 & 0.8 & 0.64 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0.64 & 0.64 & 0 & 0.8 \\
0 & 0 & 0.8 & 0.8 & 0 & 0
\end{bmatrix}
\end{array}
$$

$$\mathbf{M}$$

High-order proximity Matrix

Singular Value Decompose

$$\mathbf{M} \approx \mathbf{U\Sigma V}^\top = \sum_{i=1}^{N} \sigma_i \mathbf{u}_i \mathbf{v}_i^\top = \mathbf{D}^s \cdot \mathbf{D}^{t\top}$$

$$\mathbf{D}^s = \left[\sqrt{\sigma_1} \cdot \mathbf{u}_1, \cdots, \sqrt{\sigma_K} \cdot \mathbf{u}_K\right] = \left[\mathbf{d}_1^s, \cdots, \mathbf{d}_N^s\right]$$

$$\mathbf{D}^t = \left[\sqrt{\sigma_1} \cdot \mathbf{v}_1, \cdots, \sqrt{\sigma_K} \cdot \mathbf{v}_K\right] = \left[\mathbf{d}_1^t, \cdots, \mathbf{d}_N^t\right]$$

$$
\mathbf{D}^s = \left[\begin{array}{ccc:ccc}
-0.51 & -0.49 & -0.69 & 0 & 0 & 0 \\
0.51 & -0.49 & -0.69 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0.33 & -0.79 & 0 & 0 & 0 \\
0 & 0.71 & -0.58 & 0 & 0 & 0
\end{array}\right]
\qquad
\mathbf{D}^t = \left[\begin{array}{ccc:ccc}
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
-0.51 & 0.49 & -0.69 & 0 & 0 & 0 \\
0.51 & 0.49 & -0.69 & 0 & 0 & 0 \\
0 & -0.71 & -0.58 & 0 & 0 & 0 \\
0 & -0.33 & -0.79 & 0 & 0 & 0
\end{array}\right]
$$

Reachability between i and j

$$l_i \xrightarrow{\mathbf{d}_i^s \cdot \mathbf{d}_j^t} l_j$$

A Motivating Example

Phase (c) High-order proximity embedding

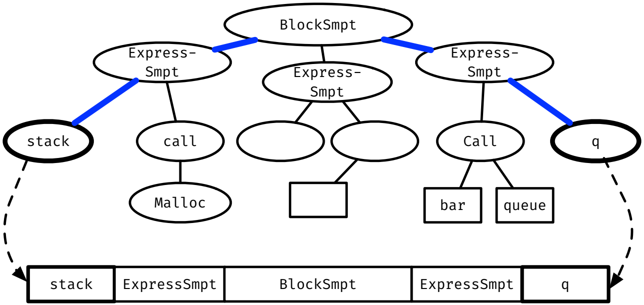
$$
\begin{array}{c}
\begin{array}{cc}
\mathbf{d}_1^s & \\
 & \\
 & \\
 & \\
 & \\
 &
\end{array}
\left[\begin{array}{ccc:ccc}
\boxed{-0.51 \quad -0.49 \quad -0.69} & & & 0 & 0 & 0 \\
0.51 & -0.49 & -0.69 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0.33 & -0.79 & 0 & 0 & 0 \\
0 & 0.71 & -0.58 & 0 & 0 & 0
\end{array}\right] \\
\mathbf{D}^s
\end{array}
$$

$$
\begin{array}{c}
\begin{array}{c}
 \\
 \\
\mathbf{d}_3^t \\
\mathbf{d}_4^t \\
\mathbf{d}_5^t \\
\mathbf{d}_6^t
\end{array}
\left[\begin{array}{ccc:ccc}
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
-0.51 & 0.49 & -0.69 & 0 & 0 & 0 \\
0.51 & 0.49 & -0.69 & 0 & 0 & 0 \\
0 & -0.71 & -0.58 & 0 & 0 & 0 \\
0 & -0.33 & -0.79 & 0 & 0 & 0
\end{array}\right] \\
\mathbf{D}^t
\end{array}
$$

Reachability	Path length
$\mathbf{d}_1^s \cdot {\mathbf{d}_5^t}^\top = 0.75$	1
$\mathbf{d}_1^s \cdot {\mathbf{d}_6^t}^\top = 0.71$	2
$\mathbf{d}_1^s \cdot {\mathbf{d}_3^t}^\top = 0.5$	3
$\mathbf{d}_1^s \cdot {\mathbf{d}_4^t}^\top = -0.02$	infeasible

A Motivating Example

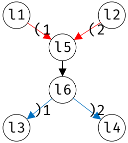
Phase (c) High-order proximity embedding



`infeasible dependence relation between `stack` to `q``

A Motivating Example

Phase (d) Value-Flow Vector and Applications



$$\left[\begin{array}{ccc|ccc} -0.51 & -0.49 & -0.69 & 0 & 0 & 0 \\ 0.51 & -0.49 & -0.69 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.33 & -0.79 & 0 & 0 & 0 \\ 0 & 0.71 & -0.58 & 0 & 0 & 0 \end{array}\right]$$

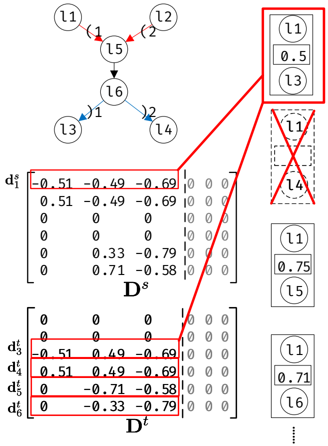
$$\mathbf{D}^s$$

$$\left[\begin{array}{ccc|ccc} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -0.51 & 0.49 & -0.69 & 0 & 0 & 0 \\ 0.51 & 0.49 & -0.69 & 0 & 0 & 0 \\ 0 & -0.71 & -0.58 & 0 & 0 & 0 \\ 0 & -0.33 & -0.79 & 0 & 0 & 0 \end{array}\right]$$

$$\mathbf{D}^t$$

A Motivating Example

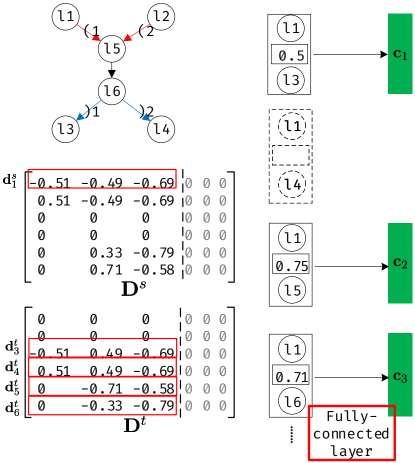
Phase (d) Value-flow Vectors and Applications



A Motivating Example

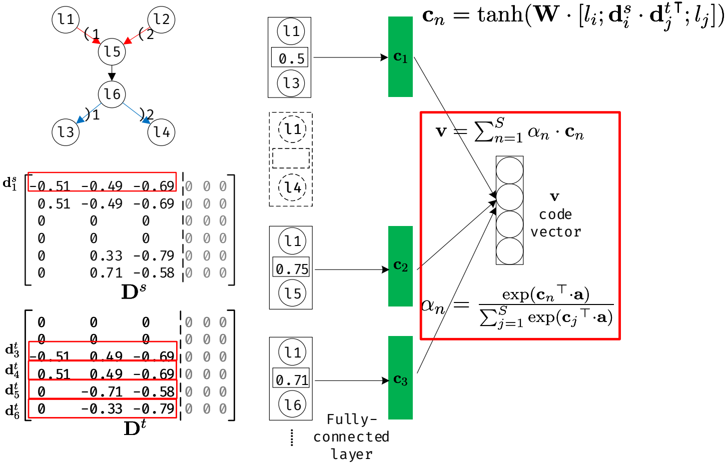
Phase (d) Value-flow Vectors and Applications

$$\mathbf{c}_n = \tanh(\mathbf{W} \cdot [l_i; \mathbf{d}_i^s \cdot \mathbf{d}_j^{t\top}; l_j])$$



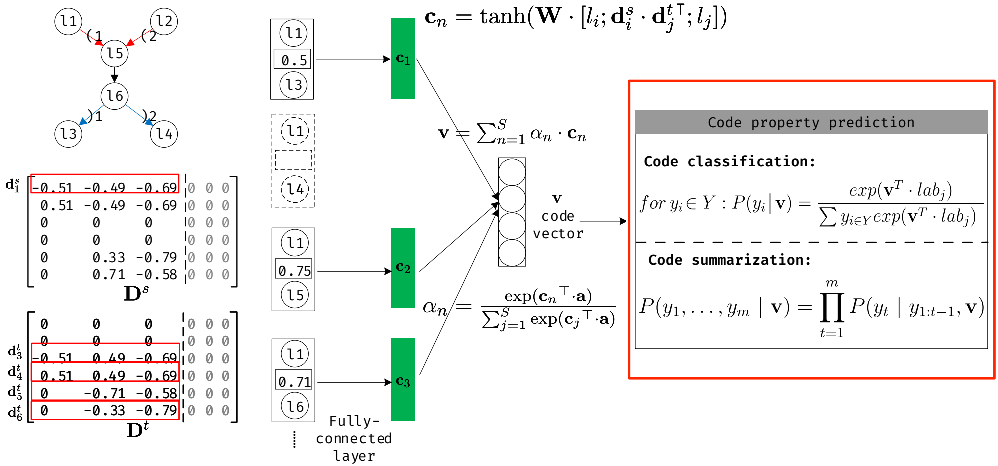
A Motivating Example

Phase (d) Value-flow Vectors and Applications



A Motivating Example

Phase (d) Value-flow Vectors and Applications



$$\mathbf{D}^s = \left[\begin{array}{ccc|ccc} -0.51 & -0.49 & -0.69 & 0 & 0 & 0 \\ 0.51 & -0.49 & -0.69 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.33 & -0.79 & 0 & 0 & 0 \\ 0 & 0.71 & -0.58 & 0 & 0 & 0 \end{array}\right]$$

$$\mathbf{D}^t = \left[\begin{array}{ccc|ccc} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -0.51 & 0.49 & -0.69 & 0 & 0 & 0 \\ 0.51 & 0.49 & -0.69 & 0 & 0 & 0 \\ 0 & -0.71 & -0.58 & 0 & 0 & 0 \\ 0 & -0.33 & -0.79 & 0 & 0 & 0 \end{array}\right]$$

Experimental Evaluation

Benchmarks

. . .

32 popular open-source C/C++ projects

Total Line of Instructions:4,922,162

Total Methods:17,529

Total Pointers: 2,913,748

Total Objects: 190,157

Total Number of Calls:536,033

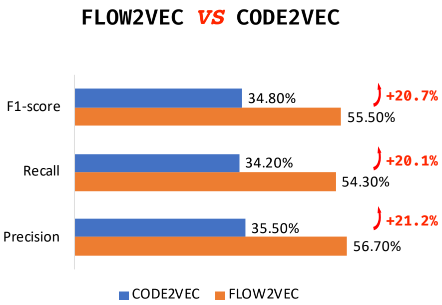
Total IVFGNodes: 4,637,301

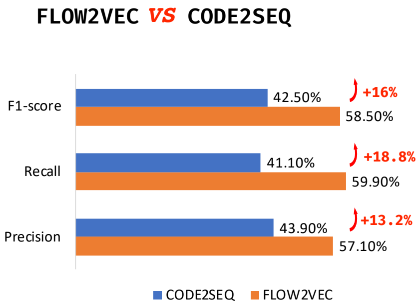
Total IVFGEdges: 6,531,578

*Conducted machine: Intel Xeon Gold 6132 @ 2.60GHz CPUs and 128GB of RAM (All finish analyzing in 272.5mins)

Experimental Evaluation

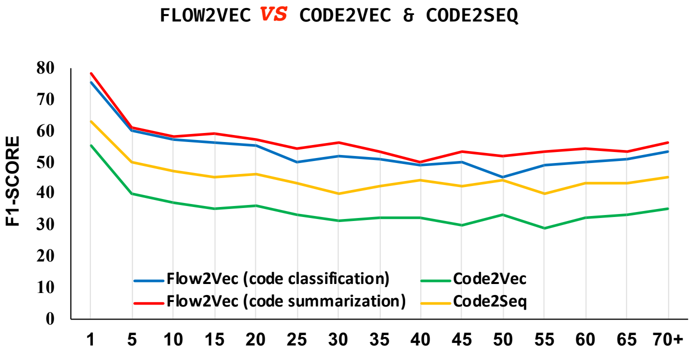
Comparison with baselines





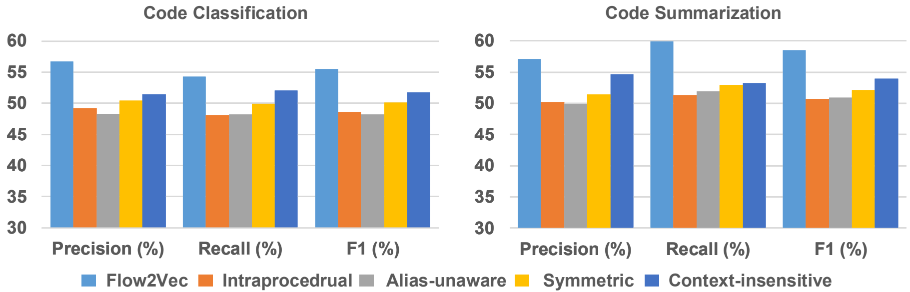
Experimental Evaluation

F1-score under different lengths of code



Experimental Evaluation

Ablation analysis



Thanks!

Q & A